Math 2500 Quiz 1 (20260128) Solution

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Total time: 10 minutes.

Formulas:

Equation for a plane: $a(x-x_0)+b(y-y_0)+c(z-z_0)=0$, where $\langle a,b,c\rangle$ is a normal vector, and (x_0,y_0,z_0) is a point on the plane.

Distance from a point $P(x_0,y_0,z_0)$ to a plane: $\frac{|\overrightarrow{QP}\cdot\mathbf{n}|}{\|\mathbf{n}\|}$, where $\mathbf{n}$ is a normal vector of the plane, and Q is a point on the plane.

Problem 1 (5 points). Find the equation of the plane containing $P(3,0,1), Q(1,-1,2), R(1,3,-1)$.

$$\overrightarrow{PQ} = \langle -2,-1,1\rangle$$
$$\overrightarrow{PR} = \langle -2,3,-2\rangle$$
$$\mathbf{n} = \overrightarrow{PQ}\times\overrightarrow{PR} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -2 & -1 & 1 \\ -2 & 3 & -2 \end{vmatrix} = \langle -1,-6,-8\rangle$$

Therefore the equation of the plane is

$$-(x-3)-6(y-0)-8(z-1)=0$$

(which simplifies to)

$$-x-6y-8z+11=0$$

Problem 2 (5 points). Find the distance between the point $P(-1,2,3)$ and the plane $2x+3y-z=4$.

A normal vector of the plane is

$$\mathbf{n} = \langle 2,3,-1\rangle$$

To find a point Q on the plane, we set $x=y=0$, and get $z=-4$. Therefore $Q(0,0,-4)$.

$$\overrightarrow{QP} = \langle -1,2,7\rangle$$
$$\text{distance} = \frac{|-2+6-7|}{\sqrt{4+9+1}} = \frac{3}{\sqrt{14}}$$