

Annulus-shaped minimizers of interaction energies have been observed in many numerical simulations, but the theoretical justification of their presence has been open for more than a decade. In this talk, I will discuss two approaches to justify that the support of a minimizer is an annulus, for the repulsive Riesz interaction $W(x) = -|x|^b/b$ with an external field. The first approach analyzes the bifurcation from a sphere-shaped minimizer for a general one-parameter family of external fields. Depending on the value of b , a sphere may bifurcate into an annulus or two spheres. The second approach starts from an explicit formula of a 'signed equilibrium', and use an iteration to eliminate its negative part and obtain the minimizer. The shape of the minimizer can be understood if certain monotone property of the signed equilibrium can be preserved through the iteration. This approach can justify the full phase transition of ball \rightarrow annulus \rightarrow sphere, although it is only applicable in limited cases of external fields.