

Section 5.6:

Mass:

For a solid Q with density $\rho(x, y, z)$:

$$m = \iiint_Q \rho(x, y, z) \, dV$$

For a lamina R with density $\rho(x, y)$:

$$m = \iint_R \rho(x, y) \, dA$$

Center of mass:

For a solid Q with density $\rho(x, y, z)$: it is $(\bar{x}, \bar{y}, \bar{z})$ where

$$\bar{x} = \frac{\iiint_Q x\rho(x, y, z) \, dV}{\iiint_Q \rho(x, y, z) \, dV}, \quad \bar{y} = \frac{\iiint_Q y\rho(x, y, z) \, dV}{\iiint_Q \rho(x, y, z) \, dV}, \quad \bar{z} = \frac{\iiint_Q z\rho(x, y, z) \, dV}{\iiint_Q \rho(x, y, z) \, dV}$$

For a lamina R with density $\rho(x, y)$: it is (\bar{x}, \bar{y}) where

$$\bar{x} = \frac{\iint_R x\rho(x, y) \, dA}{\iint_R \rho(x, y) \, dA}, \quad \bar{y} = \frac{\iint_R y\rho(x, y) \, dA}{\iint_R \rho(x, y) \, dA}$$

Moments of inertia:

For a solid Q with density $\rho(x, y, z)$:

$$I_x = \iiint_Q (y^2 + z^2)\rho(x, y, z) \, dV, \quad I_y = \iiint_Q (x^2 + z^2)\rho(x, y, z) \, dV$$

$$I_z = \iiint_Q (x^2 + y^2)\rho(x, y, z) \, dV$$

For a lamina R with density $\rho(x, y)$:

$$I_x = \iint_R y^2\rho(x, y) \, dA, \quad I_y = \iint_R x^2\rho(x, y) \, dA$$

$$I_0 = \iint_R (x^2 + y^2)\rho(x, y) \, dA = I_x + I_y$$