MATH 2500 — FORMULA SHEET FOR FINAL EXAM

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Section 2.5:

Vector equation for a line: $\mathbf{r}(t) = \mathbf{r}_0 + t\mathbf{v}$, where $\mathbf{v}$ is a direction vector, and $\mathbf{r}_0$ is the position vector of a point on the line.

Equation for a plane: $a(x - x_0) + b(y - y_0) + c(z - z_0) = 0$, where $\langle a, b, c\rangle$ is a normal vector, and (x_0, y_0, z_0) is a point on the plane.

Distance from a point P to a line: $\frac{\|\overrightarrow{PM}\times\mathbf{v}\|}{\|\mathbf{v}\|}$, where $\mathbf{v}$ is a direction vector of the line, and M is a point on the line.

Distance from a point P to a plane: $\frac{|\overrightarrow{QP}\cdot\mathbf{n}|}{\|\mathbf{n}\|}$, where $\mathbf{n}$ is a normal vector of the plane, and Q is a point on the plane.

Line of intersection between two planes: its direction vector is $\mathbf{v} = \mathbf{n}_1 \times \mathbf{n_2}$, where $\mathbf{n}_1$ and $\mathbf{n}_2$ are the normal vectors of the two planes. To find a point on the line, look for (x, y, z) satisfying the equations for both planes.

Angle between two planes: $\cos^{-1}(\frac{\mathbf{n}_1\cdot\mathbf{n_2}}{\|\mathbf{n_1}\|\cdot\|\mathbf{n_2}\|})$, where $\mathbf{n}_1$ and $\mathbf{n}_2$ are the normal vectors of the two planes.

Section 5.6:

Mass:

For a solid Q with density $\rho(x, y, z)$:

$$m = \iiint_Q \rho(x, y, z)\,\mathrm{d}V$$

For a lamina R with density $\rho(x, y)$:

$$m = \iint_R \rho(x, y)\,\mathrm{d}A$$

Center of mass:

For a solid Q with density $\rho(x, y, z)$: it is $(\bar{x}, \bar{y}, \bar{z})$ where

$$\bar{x} = \frac{\iiint_Q x\rho(x, y, z)\,\mathrm{d}V}{\iiint_Q \rho(x, y, z)\,\mathrm{d}V}, \quad \bar{y} = \frac{\iiint_Q y\rho(x, y, z)\,\mathrm{d}V}{\iiint_Q \rho(x, y, z)\,\mathrm{d}V}, \quad \bar{z} = \frac{\iiint_Q z\rho(x, y, z)\,\mathrm{d}V}{\iiint_Q \rho(x, y, z)\,\mathrm{d}V}$$

For a lamina R with density $\rho(x, y)$: it is $(\bar{x}, \bar{y})$ where

$$\bar{x} = \frac{\iint_R x\rho(x, y)\,\mathrm{d}A}{\iint_R \rho(x, y)\,\mathrm{d}A}, \quad \bar{y} = \frac{\iint_R y\rho(x, y)\,\mathrm{d}A}{\iint_R \rho(x, y)\,\mathrm{d}A}$$

Moments of inertia:
For a solid Q with density $\rho(x, y, z)$:

$$I_x = \iiint_Q (y^2 + z^2)\rho(x, y, z)\,\mathrm{d}V, \quad I_y = \iiint_Q (x^2 + z^2)\rho(x, y, z)\,\mathrm{d}V$$

$$I_z = \iiint_Q (x^2 + y^2)\rho(x, y, z)\,\mathrm{d}V$$

For a lamina R with density $\rho(x, y)$:

$$I_x = \iint_R y^2\rho(x, y)\,\mathrm{d}A, \quad I_y = \iint_R x^2\rho(x, y)\,\mathrm{d}A$$

$$I_0 = \iint_R (x^2 + y^2)\rho(x, y)\,\mathrm{d}A = I_x + I_y$$

Sections 6.4, 6.7, 6.8:

Green's Theorem (circulation form): Let C be a simple closed curve in 2D, traveling counterclockwise, enclosing the region D. Let $\mathbf{F} = \langle P, Q\rangle$ be a vector field. Then

$$\oint_C \mathbf{F}\cdot\,\mathrm{d}\mathbf{r} = \iint_D (\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y})\,\mathrm{d}A$$

Green's Theorem (flux form): Let C be a simple closed curve in 2D, with outward normal, enclosing the region D. Let $\mathbf{F}$ be a vector field. Then

$$\oint_C \mathbf{F}\cdot\mathbf{N}\,\mathrm{d}s = \iint_D \nabla\cdot\mathbf{F}\,\mathrm{d}A$$

Stokes' Theorem: Let S be an oriented surface whose boundary is a simple closed curve C with positive orientation. Let $\mathbf{F}$ be a vector field. Then

$$\oint_C \mathbf{F}\cdot\,\mathrm{d}\mathbf{r} = \iint_S (\nabla\times\mathbf{F})\cdot\mathbf{N}\,\mathrm{d}S$$

Divergence Theorem: Let S be a closed surface, with outward normal, enclosing the 3D region E. Let $\mathbf{F}$ be a vector field. Then

$$\iint_S \mathbf{F}\cdot\mathbf{N}\,\mathrm{d}S = \iiint_E \nabla\cdot\mathbf{F}\,\mathrm{d}V$$