MATH 2500 QUIZ 10 SOLUTION (20260422)
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Total time: 10 minutes.

Problem 1 (5 points). Let S be the part of the plane $2x + y - z = 1$ with $x^2 + y^2 \leq 1$. Parametrize S. (Hint: S is part of a graph)

$$\mathbf{r}(u, v) = \langle u, v, 2u + v - 1 \rangle, \quad (u, v) \in D = \{(u, v) : u^2 + v^2 \leq 1\}$$

Problem 2 (5 points). Use your result in Problem 1 to write the following surface integral as a double integral (where $\mathbf{N}$ is the downward normal). DO NOT calculate it.

$$\iint_S \langle 0, y^2, 2x \rangle \cdot \mathbf{N} \, \mathrm{d}S$$

$$\mathbf{r}_u = \langle 1, 0, 2 \rangle, \quad \mathbf{r}_v = \langle 0, 1, 1 \rangle$$

$$\mathbf{r}_u \times \mathbf{r}_v = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & 0 & 2 \\ 0 & 1 & 1 \end{vmatrix} = \langle -2, -1, 1 \rangle$$

Since $\mathbf{N}$ is the downward normal, the third component of $\mathbf{N}$ is negative, thus in the opposite direction of $\mathbf{r}_u \times \mathbf{r}_v$.

$$\iint_S \langle 0, y^2, 2x \rangle \cdot \mathbf{N} \, \mathrm{d}S = -\iint_D \langle 0, v^2, 2u \rangle \cdot \langle -2, -1, 1 \rangle \, \mathrm{d}A = -\iint_D (-v^2 + 2u) \, \mathrm{d}A$$