MATH 2500 QUIZ 7 SOLUTION (20260401)
INSTRUCTOR: RUIWEN SHU

Total time: 10 minutes.

Problem 1 (6 points). Let C be the segment connecting $P(1,0,1)$ and $Q(-1,2,0)$. Calculate the following scalar line integral:

$$\int_C (x+z)\,\mathrm{d}s$$

Parametrization of C:

$$\mathbf{r}(t) = \langle 1,0,1\rangle + t\langle -2,2,-1\rangle = \langle 1-2t, 2t, 1-t\rangle, \quad 0 \le t \le 1$$

$$\mathbf{r}'(t) = \langle -2,2,-1\rangle, \quad \|\mathbf{r}'(t)\| = \sqrt{4+4+1} = 3$$

Therefore

$$\int_C (x+z)\,\mathrm{d}s = \int_0^1 (1-2t+1-t)\cdot 3\,\mathrm{d}t = 3\int_0^1 (2-3t)\,\mathrm{d}t = 3(2t-\frac{3}{2}t^2)\Big|_0^1 = \frac{3}{2}$$

Problem 2 (4 points). Let C be the upper half of the circle $x^2+y^2=4$. Write a definite integral to represent

$$\int_C x\sin y\,\mathrm{d}s$$

You DO NOT need to calculate this integral.
Parametrization of C:

$$\mathbf{r}(t) = \langle 2\cos t, 2\sin t\rangle, \quad 0 \le t \le \pi$$

$$\mathbf{r}'(t) = \langle -2\sin t, 2\cos t\rangle, \quad \|\mathbf{r}'(t)\| = 2$$

Therefore

$$\int_C x\sin y\,\mathrm{d}s = \int_0^\pi 2\cos t\cdot\sin(2\sin t)\cdot 2\,\mathrm{d}t$$