Total time: 10 minutes.

Problem 1 (6 points). Let C be the part of the curve $y = x^3$ between $x = 0$ and $x = 1$, traveling from left to right. Calculate the following vector line integral:

$$\int_C \langle xy, y\rangle \cdot \,\mathrm{d}\mathbf{r}$$

This curve is a graph, parametrized by

$$\mathbf{r}(t) = \langle t, t^3\rangle, \quad 0 \le t \le 1$$

$$\mathbf{r}'(t) = \langle 1, 3t^2\rangle$$

$$\int_C \langle xy, y\rangle \cdot \,\mathrm{d}\mathbf{r} = \int_0^1 \langle t \cdot t^3, t^3\rangle \cdot \langle 1, 3t^2\rangle \,\mathrm{d}t = \int_0^1 (t^4 + 3t^5)\,\mathrm{d}t = (\frac{1}{5}t^5 + \frac{1}{2}t^6)\Big|_0^1 = \frac{7}{10}$$

Problem 2 (4 points). Show that the following vector field is not conservative:

$$\mathbf{F}(x, y, z) = \langle x^2 + yz,\ xz - 2yz,\ x^2y + x^3z^2\rangle$$

Denote the components of $\mathbf{F}$ as P, Q, R. Use cross derivative test:

$$\frac{\partial P}{\partial y} = z, \quad \frac{\partial Q}{\partial x} = z$$

$$\frac{\partial P}{\partial z} = y, \quad \frac{\partial R}{\partial x} = 2xy + 3x^2z^2$$

Since $\frac{\partial P}{\partial z} \neq \frac{\partial R}{\partial x}$, we see that $\mathbf{F}$ is not conservative.